Problem Set 10Q1 + 2019ex Q2

Macroeconomics III

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Problem 1

Consider a calvo economy with producers whose ideal prices are given by,

$$p_t^* = \phi m_t + (1-\phi)p_t$$

- ϕ measures the amount of rigidity.
- Firms can change prices with probability α in each period
- Households discount the future by β

Inflation is given by,

$$\pi_t = p_t - p_{t-1}$$

The New-Keynesian Philips Curve holds and is given by,

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}]$$

where $\kappa = \frac{\alpha\phi[1-\beta(1-\alpha)]}{1-\alpha}$

Finally, from the quantity theory we have,

$$y_t = m_t - p_t$$

Problem 1a - Conjecture of price level

We conjecture that the behaviour of the price level is described by

$$p_t = \left(1 - \lambda^t\right) m_1$$

Explain why this conjecture is or is not reasonable.

We know that only a fraction α can change their prices in each period, so prices will not adjust to the new equilibrium instantly. The fraction of firms that can change their prices is constant, so it seems reasonable that the difference between the current prices and the equilibrium prices, m_1, is reduced by a constant relative amount. Hence, the conjecture seems reasonable.

Problem 1b

Find λ in terms of κ and β

In total we have the following equations,

$$p_t^* = \phi m_t + (1-\phi)p_t \tag{1}$$
$$\pi_t = p_t - p_{t-1} \tag{2}$$
$$\pi_t = \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}] \tag{3}$$
$$y_t = m_t - p_t \tag{4}$$
$$p_t = \left(1-\lambda^t\right) m_1 \tag{5}$$

1. Combine equations to get y_t, π_t and $\mathbb{E}_t[\pi_{t+1}]$ in terms of λ and m_1.
2. Insert into (3) and isolate λ
3. Rewrite to get a second order polynomial in λ and solve it
4. Use that $|\lambda| < 1$ (Why?). Check that it holds.

Problem 1b - Finding π_t, $\mathbb{E}_t[\pi_{t+1}]$ and y_t

We combine (2) and (5),

$$\begin{aligned}
\pi_t &= p_t - p_{t-1} \\
&= \left(1 - \lambda^t\right) m_1 - \left(1 - \lambda^{t-1}\right) m_1 \\
&= \left(\lambda^{t-1} - \lambda^t\right) m_1 \\
&= (1 - \lambda)\lambda^{t-1} m_1
\end{aligned}$$

We lead inflation one period and take expectations

$$\begin{aligned}
E_t\left[\pi_{t+1}\right] &= E_t\left[(1 - \lambda)\lambda^t m_1\right] \\
&= (1 - \lambda)\lambda^t m_1
\end{aligned}$$

We then combine (5) and (4)

$$\begin{aligned}
y_t &= m_1 - p_t \\
&= m_1 - \left(1 - \lambda^t\right) m_1 \\
&= \lambda^t m_1
\end{aligned}$$

Problem 1b - Insert into NKPC

We insert the results into the NKPC,

$$\begin{aligned}
\pi_t &= \kappa y_t + \beta \mathbb{E}_t[\pi_{t+1}] \\
\underbrace{(1-\lambda)\lambda^{t-1} m_1}_{\pi_t} &= \kappa \underbrace{\lambda^t m_1}_{y_t} + \beta \underbrace{(1-\lambda)\lambda^t m_1}_{E_t[\pi_{t+1}]} \\
1-\lambda &= \kappa\lambda + \beta(1-\lambda)\lambda \\
1-\lambda &= \kappa\lambda + \beta\lambda - \beta\lambda^2 \\
0 &= \beta\lambda^2 - (1+\kappa+\beta)\lambda + 1
\end{aligned}$$

We now have a second order polynomial. We solve it by the quadratic equation:

$$\lambda = \frac{1+\kappa+\beta \pm \sqrt{(1+\kappa+\beta)^2 - 4\beta}}{2\beta}$$

Problem 1b - Finding λ

$$\lambda = \frac{1+\kappa+\beta \pm \sqrt{(1+\kappa+\beta)^2 - 4\beta}}{2\beta}$$

We look at the term in the square root to check if there are real roots,

$$\begin{aligned}(1+\kappa+\beta)^2 - 4\beta &= 1+\kappa^2+\beta^2+2\kappa\beta+2\kappa+2\beta-4\beta \\ &= 1+\kappa^2+\beta^2+2\kappa\beta+2\kappa-2\beta \\ &= (1-\beta)^2+\kappa^2+2\kappa\beta+2\kappa > 0\end{aligned}$$

Hence, there must be two real roots.

For the economy to be stable it must be that $|\lambda| < 1$ such that $p_t \to m_1$ (look at the conjecture and see what happens when $t \to \infty$ if $|\lambda| > 1$).

We know that $\beta < 1$ and thus $\frac{1+\kappa+\beta}{2\beta} > 1$. For $|\lambda| < 1$ to hold, it must be a minus sign, why the solution is,

$$\lambda = \frac{1+\kappa+\beta - \sqrt{(1+\kappa+\beta)^2 - 4\beta}}{2\beta}$$

Problem 1c - Effect on λ from ϕ and α (1/3)

How does increases in each of ϕ, α and β affect λ? Explain.

$$\lambda = \frac{1+\kappa+\beta-\sqrt{(1+\kappa+\beta)^2-4\beta}}{2\beta}$$
$$\kappa = \frac{\alpha\phi[1-\beta(1-\alpha)]}{1-\alpha}$$

Using the chain-rule the derivatives wrt. ϕ and α are,

$$\frac{\partial\lambda}{\partial\phi} = \frac{\partial\lambda}{\partial\kappa}\cdot\frac{\partial\kappa}{\partial\phi}$$
$$\frac{\partial\lambda}{\partial\alpha} = \frac{\partial\lambda}{\partial\kappa}\cdot\frac{\partial\kappa}{\partial\alpha}$$

Looking at the expression for κ, we see

$$\frac{\partial\kappa}{\partial\phi} > 0 \qquad \text{and} \qquad \frac{\partial\kappa}{\partial\alpha} > 0$$

Problem 1c - Effect on λ from ϕ and α (2/3)

Next, we look at the derivative of λ wrt. κ.

$$\lambda = \frac{1+\kappa+\beta-\sqrt{(1+\kappa+\beta)^2-4\beta}}{2\beta}$$

Taking the derivative yields,

$$\frac{\partial\lambda}{\partial\kappa} = \frac{1}{2\beta}\underbrace{\left(1-\frac{1+\kappa+\beta}{\sqrt{(1+\kappa+\beta)^2-4\beta}}\right)}_{<0}$$

The parentheses is less than zero since,

$$1+\kappa+\beta = \sqrt{(1+\kappa+\beta)^2} > \sqrt{(1+\kappa+\beta)^2-4\beta}$$

Problem 1c - Effect on λ from ϕ and α (3/3)

$$\frac{\partial \lambda}{\partial \phi} = \frac{\partial \lambda}{\partial \kappa} \cdot \frac{\partial \kappa}{\partial \phi} < 0$$

Intuition: If α increases, the amount of firms that can adjust their prices increases. Hence, the speed of convergence towards the new steady state increases, which corresponds to lower λ.

$$\frac{\partial \lambda}{\partial \alpha} = \frac{\partial \lambda}{\partial \kappa} \cdot \frac{\partial \kappa}{\partial \alpha} < 0$$

Intuition: High ϕ corresponds to a low degree of frictions. If ϕ increases, the nominal rigidities decreases, which increases the speed of convergence.

Problem 1c - Effect on λ from β (1/2)

How does increases in each of ϕ, α and β affect λ? Explain.

$$\lambda = \frac{1+\kappa+\beta-\sqrt{(1+\kappa+\beta)^2-4\beta}}{2\beta}$$

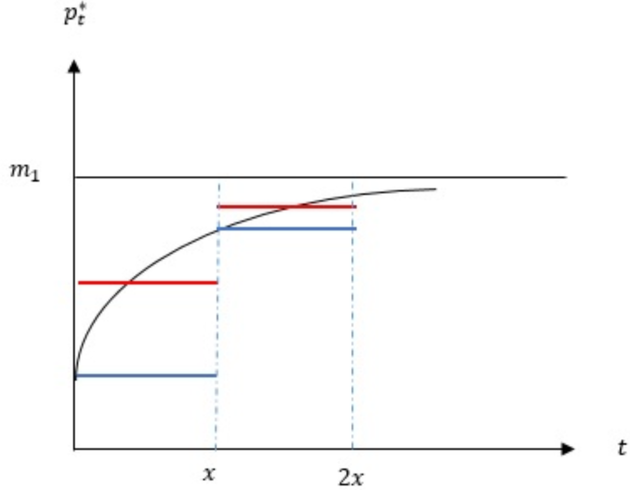
$$\kappa = \frac{\alpha\phi[1-\beta(1-\alpha)]}{1-\alpha}$$

We can see that β affects λ both directly and through κ.

The overall effect is $\frac{\partial\lambda}{\partial\beta} < 0$

As m increases, the firms face a trade-off between short and long term profits. Suboptimally low profits now maximize short term profits but lowers profits later on. If β increases, firms become more patient so they value future profits more and will set their prices higher in response to $m_1 > m$. This lowers λ and increases speed of convergence

Problem 1c - Effect on λ from β (2/2)



January 2019 Q2

Representative agent with utility function

$$U_i = C_i - \frac{1}{\beta} L_i^{\beta}, \quad \beta > 1 \tag{6}$$

With budget constraint

$$PC_i = P_i Y_i \tag{7}$$

The production function is given by

$$Y_i = L_i^{\alpha}, \quad 0 < \alpha < 1 \tag{8}$$

There is monopolistic competition with demand for good i:

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y \tag{9}$$

Aggregate demand is given by

$$Y = \frac{M}{P} \tag{10}$$

January 2019 Q2

Derive the optimal production y_i^* as a function of the relative price $p_i - p$ after taking logs of the FOC from the utility maximization problem,

$$U_i = C_i - \frac{1}{\beta} L_i^{\beta}, \quad \beta > 1 \tag{11}$$

$$PC_i = P_i Y_i \tag{12}$$

$$Y_i = L_i^{\alpha}, \quad 0 < \alpha < 1 \tag{13}$$

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y \tag{14}$$

- Insert (12), (13), (14) into (11) to get $U_i = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\beta} Y_i^{\frac{\beta}{\alpha}}$
  Find the FOC of U_i wrt. Y_i
- Insert P_i and P back into the equation by using (14).
- Take logs and find y_i as a function of p_i, p and parameters.

Remember that: $Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y \implies \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} = \left(\frac{P_i}{P}\right)$

January 2019 Q2 - Question a (1/3)

First, we isolate C_i in (12)

$$C_i = \frac{P_i}{P} Y_i$$

Next, we isolate L_i in (13)

$$L_i = Y_i^{\frac{1}{\alpha}}$$

Finally, we isolate $\frac{P_i}{P}$ in (14)

$$Y_i = \left(\frac{P_i}{P}\right)^{-\eta} Y \quad \Longrightarrow \quad \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} = \left(\frac{P_i}{P}\right)$$

We insert the results into the utility function

$$U_i = C_i - \frac{1}{\beta} L_i^{\beta} = \frac{P_i}{P} Y_i - \frac{1}{\beta} Y_i^{\frac{\beta}{\alpha}} = \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\beta} Y_i^{\frac{\beta}{\alpha}}$$

January 2019 Q2 - Question a (2/3)

$$
\begin{aligned}
U_i &= \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} Y_i - \frac{1}{\beta} Y_i^{\frac{\beta}{\alpha}} \\
&= \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} Y_i^{1-\frac{1}{\eta}} - \frac{1}{\beta} Y_i^{\frac{\beta}{\alpha}}
\end{aligned}
$$

First-order condition wrt. Y_i implies:

$$
\begin{aligned}
\frac{\partial U_i}{\partial Y_i} = 0 \quad \Longrightarrow \quad \left(1 - \frac{1}{\eta}\right) \cdot \left(\frac{1}{Y}\right)^{-\frac{1}{\eta}} \cdot Y_i^{-\frac{1}{\eta}} &= \frac{1}{\beta} \cdot \frac{\beta}{\alpha} Y_i^{\frac{\beta}{\alpha}-1} \\
\left(1 - \frac{1}{\eta}\right) \cdot \left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} &= \frac{1}{\alpha} Y_i^{\frac{\beta-\alpha}{\alpha}}
\end{aligned}
$$

Next, we insert $\left(\frac{Y_i}{Y}\right)^{-\frac{1}{\eta}} = \left(\frac{P_i}{P}\right)$,

$$
\left(1 - \frac{1}{\eta}\right) \cdot \left(\frac{P_i}{P}\right) = \frac{1}{\alpha} Y_i^{\frac{\beta-\alpha}{\alpha}}
$$

January 2019 Q2 - Question a (3/3)

Now, we have the following equation

$$\left(1-\frac{1}{\eta}\right)\cdot\left(\frac{P_i}{P}\right)=\frac{1}{\alpha}Y_i^{\frac{\beta-\alpha}{\alpha}}$$

Then we take the log of the the equation and use the following notation $\ln(X)=x$

$$\begin{aligned}
p_i-p+\ln\left(1-\frac{1}{\eta}\right)&=\frac{\beta-\alpha}{\alpha}y_i+\ln\left(\frac{1}{\alpha}\right)\\
y_i\frac{\beta-\alpha}{\alpha}&=p_i-p+\ln\left(1-\frac{1}{\eta}\right)-\ln\left(\frac{1}{\alpha}\right)\\
y_i&=\frac{\alpha}{\beta-\alpha}(p_i-p)+\frac{\alpha}{\beta-\alpha}\left[\ln\left(1-\frac{1}{\eta}\right)-\ln\left(\frac{1}{\alpha}\right)\right]
\end{aligned}$$

January 2019 Q2 - Question b (1/2)

Impose homogeneity and show that $y = \frac{\alpha}{\beta-\alpha} \ln \left(\alpha \frac{\eta-1}{\eta} \right)$. Provide an economic interpretation to this result.

We impose homogeneity $p_i = p$ such that

$$
\begin{aligned}
y_i &= \frac{\alpha}{\beta-\alpha}(p_i - p) + \frac{\alpha}{\beta-\alpha}\left[\ln\left(1-\frac{1}{\eta}\right) - \ln\left(\frac{1}{\alpha}\right)\right] \\
&= \frac{\alpha}{\beta-\alpha}\left[\ln\left(1-\frac{1}{\eta}\right) - \ln\left(\frac{1}{\alpha}\right)\right] \\
&= \frac{\alpha}{\beta-\alpha}\ln\left(\alpha\frac{\eta-1}{\eta}\right)
\end{aligned}
$$

We then show that y is indeed increasing in η by taking the derivative wrt. η,

$$
\frac{\partial y}{\partial \eta} = \frac{\alpha}{\beta-\alpha}\frac{1}{\eta(\eta-1)} > 0
$$

It is positive since $\eta > 1$ and $\beta > \alpha$.

January 2019 Q2 - Question b (2/2)

We have the following,

$$\frac{\partial y}{\partial \eta} = \frac{\alpha}{\beta - \alpha} \frac{1}{\eta(\eta - 1)} > 0$$

What is the intuition?

η measures the degree to which the goods in the monopolistically competitive market are substitutable. So if η increases, the goods become more substitutable, which makes the deadweight loss lower and output higher. If the monopolistic firms are closer to being competitors, the loss from imperfect competition gets lower.

January 2019 Q2 - Question c (1/2)

Suppose now that individual prices are fixed for 2 periods, and that price-setting is staggered, such that $\frac{1}{2}$ of the prices are set in period t at the level x_t, and $\frac{1}{2}$ were set in period $t+1$ at the level x_{t+1}. Thus, the aggregate price level equals

$$p_t = \frac{1}{2}(x_t + x_{t+1})$$

Assuming certainty equivalence (i.e., $x_t = \frac{1}{2}(p^*_{i,t} + \mathbb{E}_t[p^*_{i,t+1}])$, where $p^*_{i,t} = m_t + y$ denotes the optimal reset price), show that the equilibrium reset price, x_t, depends on m_t and $\mathbb{E}_t[m_{t+1}]$).

January 2019 Q2 - Question c (2/2)

Show that x_t depends on m_t and $\mathbb{E}_t[m_{t+1}]$)

$$
\begin{aligned}
p_t &= \frac{1}{2}(x_t + x_{t+1}) \\
x_t &= \frac{1}{2}(p^*_{i,t} + \mathbb{E}_t[p^*_{i,t+1}]) \\
p^*_{i,t} &= m_t + y
\end{aligned}
$$

We combine the equations,

$$
\begin{aligned}
x_t &= \frac{1}{2}(p^*_{i,t} + \mathbb{E}_t[p^*_{i,t+1}]) \\
&= \frac{1}{2}(m_t + y + \mathbb{E}_t[m_{t+1} + y]) \\
&= \frac{1}{2}(m_t + 2 \cdot y + \mathbb{E}_t[m_{t+1}) \\
&= y + \frac{1}{2}(m_t + \mathbb{E}_t[m_{t+1})
\end{aligned}
$$

Higher contemporaneous or expected money supply m increases the desired price and thus x_t.